MEDICAPS UNIVERSITY

Ph. D Written Test Format and Syllabus

Mathematics, Faculty of Science

Ph.D. Admission Test Format

The written test consists of two parts.

1. **Part A:** Research Methodology 25 questions.
2. **Part B**: Mathematics 25 questions

Part A: Research Methodology Syllabus

Research Fundamentals:

Meaning of research; objectives of research; characteristics of good research, Research problem: Identification, selection, and techniques for defining research problem, Research process, Research outcomes, Review of Literature, Hypothesis: Definition and Types

Types of Research:

Types of research, fundamental and applied research, qualitative and quantitative. Research Design: Types of research design – Exploratory, Descriptive, Casual Analytical

Sampling, Data Collection and analysis:

Types and sources of data: Primary and secondary, Methods of collecting data: questionnaire, interview, observation, case study, experiments etc., Sampling and sampling methods, characteristics of good sample, sampling techniques, Statistical Methods for Data Analysis: measures of central tendency and dispersion

Research Report:

Main body of report, abstract and keywords, Referencing styles and bibliography. Journal and author indexing

Ethics in Research:

Biasing: Definition and Types, Plagiarism -Definition and forms, IPR, copyright infringement, AI Generated Content

Part B: Mathematics Syllabus

Real Analysis and Complex Analysis:

Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence, Improper Integrals, partial derivative.

Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Algebra and Linear Algebra:

Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups.

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions.

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Ordinary Differential Equations (ODEs) and Partial Differential Equations:

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs.

General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Numerical Analysis and Integral transform and Equations:

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Laplace and Fourier Transform. Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Descriptive statistics, exploratory data analysis

Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).

Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.

Standard discrete and continuous univariate distributions. sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range.

Methods of estimation, properties of estimators, confidence intervals. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests.