Medi-Caps University

Syllabus for Ph. D. Entrance Exam in Mathematics

Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals.

Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems.

Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations.Algebra of matrices, rank and determinant of matrices linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem.Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions.Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Algebra: Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups.

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions.

Ordinary Differential Equations (ODEs):

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs.

General theory of homogenous and non-homogeneous linear ODEs, variation of parameters,Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs):

Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Numerical Analysis :

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Integral transform and Equations:

Laplace and Fourier Transform.

Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Descriptive statistics, exploratory data analysis

Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).

Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.

Standard discrete and continuous univariate distributions. sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range.

Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests.

Simple nonparametric tests for one and two sample problems, rank correlation and test for independence. Elementary Bayesian inference.

Gauss-Markov models, estimability of parameters, best linear unbiased estimators, confidence intervals, tests for linear hypotheses. Analysis of variance and covariance. Fixed, random and mixed effects models. Simple and multiple linear regression.

Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling. Ratio and regression methods.

Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD. 2^k factorial experiments: confounding and construction.

Hazard function and failure rates, censoring and life testing, series and parallel systems.

Linear programming problem, simplex methods, duality. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.