Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Science

End Sem (Even) Examination May-2018

CA3CO08 Mathematics-II

Programme: BCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Rolle's theorem will be verified for the function $f(x) = x^2 - 6x + 8$ in the interval [2, 4] when the value of x is **1**

(a) 2 (b) 3 (c) 4 (d) 0

ii. The value of $D^n\ (ax+b)^m$ is equal to **1**

(a) $\frac{m!}{(m-n)!}a^n(ax+b)^{m-n}$ (b) $m!\,a^n\ (ax+b)^m$

(c) $\frac{1}{(m-n)!}(ax+b)^{m-n}$ (d) $m!\,(ax+b)^{m-n}$

iii. The condition for the point (a, b) to be a saddle point of the function $f(x, y)$ is **1**

(a) $rt - s^2 = 0$ (b) $rt - s^2 < 0$

(c) $rt - s^2 > 0$ (d) None of these

iv. If $u = x^3 + y^3 - 3xy^2$ then value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to **1**

(a) u (b) 3u (c) 2u (d) 1

v. $\int_0^{\pi/2} sin^{2m-1}\theta\, cos^{2n-1}\theta d\theta =$ **1**

(a) $B(m, n)$ (b) $B(n, m)$ (c) $\frac{1}{2}B(m, n)$ (d) $2B(m, n)$

vi. $\Gamma(n+1) =$, where n is an integer. **1**

(a) $n!$ (b) 1 (c) $(n-1)!$ (d) 0

vii. $\int_0^1\int_0^1 xy\ dxdy =$ **1**

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) 0

P.T.O.

viii. $\int_0^{\pi/2}\int_{\pi/2}^{\pi}\sin(x+y)\,dydx=$ **1**

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) -1

ix. The condition that $Mdx + Ndy = 0$ to be exact is **1**

(a) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$ (b) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ (c) $\frac{\partial M}{\partial x}=2\frac{\partial N}{\partial y}$ (d) $2\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$

x. General solution of differential equation $\frac{d^2y}{dx^2}-y=0$ is **1**

(a) $y=e^x$ (b) $y=e^{-x}$

(c) $y=ae^x+be^{-x}$ (d) None of these

Q.2 i. Verify first mean value theorem for the function $f(x)=x^3-3x^2+2x+5$ in [0, 1]. **4**

ii. Expand $\tan^{-1}x$ in ascending powers of x by Maclaurin's theorem. **6**

OR iii. If $y=(\sin^{-1}x)^2$ then prove that **6**

$$(1-x^2)y_{n+2}-(2x+1)xy_{n+1}-n^2y_n=0.$$

Q.3 Attempt any two:

i. If $u=\log(x^3+y^3+z^3-3xyz)$ than prove that **5**

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$$

ii. Discuss the maximum and minimum values of the following function **5**

$$u=xy+\frac{a^3}{x}+\frac{a^3}{y}$$

iii. State and prove Euler's theorem for function of two variables. **5**

Q.4 i. Prove that $\Gamma\,1/2=\sqrt{\pi}$ **3**

ii. Prove that $\beta(m,n)=\frac{\Gamma m\Gamma n}{\Gamma(m+n)}$ **7**

OR iii. Show that $\int_0^2 x\,(8-x^3)^{1/3}dx=\frac{16\pi}{9\sqrt{3}}$. **7**

Q.5 i. Evaluate **4**

$$\int_0^3\int_0^2\int_0^1(x+y+z)\,dxdydz.$$

ii. Evaluate **6**

$$\int_0^1\int_0^{\sqrt{(1+x^2)}}\frac{dxdy}{1+x^2+y^2}$$

OR iii. Evaluate $\iint_R y\,dxdy$ where R is the region bounded by the parabola $y^2=4x$ & $x^2=4y$. **6**

Q.6 i. Solve **3**

$$(1+4xy+2y^2)\,dx+(1+4xy+2x^2)\,dy=0$$

ii. Solve **7**

$$(1+y^2)+(x-e^{-\tan^{-1}y})\frac{dy}{dx}=0$$

OR iii. Solve **7**

$$(D^2-2D+5)\,y=e^{2x}\sin x$$

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MEDICAPS UNIVERSITY, INDORE
END SEM EXAMINATION MAY-2018
CA3CO08 MATHEMATICS – II
B.C.A. (EVEN SEM)

Max. Mark

Que.1(i) (b) 3 — 1

(ii) (a) $\frac{m!}{(m-n)!} a^n (ax+b)^{m-n}$ — 1

(iii) (b) $rt - s^2 < 0$ — 1

(iv) (b) $3u$ — 1

(v) (c) $\frac{1}{2} B(m,n)$ — 1

(vi) (a) $n!$ — 1

(vii) (c) $\frac{1}{4}$ — 1

(viii) (a) 0 — 1

(ix) (b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ — 1

(x) (c) $Y = ae^{x} + be^{-x}$. — 1

Que.2(i) Given $f(x) = x^3 - 3x^2 + 2x + 5$ in $[0,1]$.

By Lagrange's Mean value theorem or first mean value theorem we know that "If $f(x)$ be a function defined on $[a,b]$ such that (i) $f(a) \neq f(b)$ (ii) $f(x)$ is continuous function in the closed interval $[a,b]$, (iii) $f(x)$ is differentiable in the open interval (a,b). Then there exists at least one real value $c \in (a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{(b-a)}\text{"}$$ +2

Clearly here we have $f(0) = 5$ and $f(1) = 1-3+2+5 = 5$ +1

i.e. $f(0) = f(1) = 5$

Also $f'(x) = 3x^2 - 6x + 2 \Rightarrow f'(c) = 3c^2 - 6c + 2$

$\therefore$ By L.M.V.T.

$$f'(c) = \frac{f(b)-f(a)}{b-a} \Rightarrow 3c^2 - 6c + 2 = 0$$

$$c = \frac{-(-6) \pm \sqrt{(6)^2 - 4 \times 3 \times 2}}{2 \times 3} = \frac{6 \pm \sqrt{12}}{6}$$

i.e. $c \notin [0,1]$

Hence we see that the given function does not satisfies the conditions of L.M.V.T. Hence L.M.V.T. or first mean value theorem can ~~be~~ not be verified. +1

Que. 2(ii) Let $y = f(x) = \tan^{-1}x \longrightarrow (1)$

$\Rightarrow y_0 = \tan^{-1}0 = 0$ +1

Differentiating equ. (1) w.r.t 'x' we get

$y_1 = \frac{1}{(1+x^2)} = (1+x^2)^{-1}$

or $y_1 = 1 - x^2 + x^4 - x^6 + \cdots$ ($\because$ By Binomial expansion $(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$)

$\Rightarrow (y_1)_0 = 1$ +1

By successive differentiation of y_1, w.r.t 'x' we get

$y_2 = -2x + 4x^3 - 6x^5 + \cdots$

$(y_2)_0 = 0$ +1

$y_3 = -2 + 12x^2 - 30x^4 + \cdots$

$(y_3)_0 = -2$ +1

$y_4 = 24x - 120x^3 + \cdots$

$(y_4)_0 = 0$

$y_5 = 24 - 360x^2 + \cdots$

$(y_5)_0 = 24$

We know that by McLaurin's theorem

$y = y_0 + \frac{x}{\underline{|1}}(y_1)_0 + \frac{x^2}{\underline{|2}}(y_2)_0 + \frac{x^3}{\underline{|3}}(y_3)_0 + \frac{x^4}{\underline{|4}}(y_4)_0 + \frac{x^5}{\underline{|5}}(y_5)_0 + \cdots$ +1

$\therefore y = \tan^{-1}x = 0 + \frac{x}{\underline{|1}}1 + \frac{x^2}{\underline{|2}}0 + \frac{x^3}{\underline{|3}}(-2) + \frac{x^4}{\underline{|4}} \times 0 + \frac{x^5}{\underline{|5}}24 + \cdots$

$\therefore y = \tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$ +1

e. 2(iii) Given $y = (\sin^{-1}x)^2 \longrightarrow (1)$

Differentiating equ. (1) w.r.t 'x' we get

$y_1 = 2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}}$ or $\sqrt{1-x^2}\, y_1 = 2\sin^{-1}x$ +1

squaring both ~~sides~~ we get

$(1-x^2)\, y_1^2 = 4(\sin^{-1}x)^2 \Rightarrow (1-x^2)y_1^2 = 4y \longrightarrow (2)$ ($\because$ by (1)) +0.5

Again diff. equ. (2) w.r.t 'x' we get

$(1-x^2)\, 2y_1 \cdot y_2 + (0-2x)\, y_1^2 = 4y_1$

or $(1-x^2)y_2 - xy_1 = 2 \longrightarrow (3)$ (dividing by $2y_1$ +1.5

Differentiating equ. (3) n times by Leibnitz's theorem*, we get +1

$$D^n((1-x^2)y_2) - D^n(xy_1) = D^n(2)$$ +1

$$\left[(1-x^2).y_{n+2} + n.(-2x)y_{n+1} + \frac{n(n-1)}{\lfloor 2}(-2)y_n\right]$$

$$-\left[x.y_{n+1} + n.(1).y_n\right] = 0$$

$$\Rightarrow (1-x^2)y_{n+2} - 2nxy_{n+1} - (n^2-n)y_n - xy_{n+1} - ny_n = 0$$

Thus we have $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0.$ +1

*(Leibnitz's theorem " If u and v are function of x, then

$$D^n(u.v) = {}^nC_0 D^n u + {}^nC_1 D^{n-1}u . Dv + {}^nC_2 D^{n-2}u . D^2v + \cdots + {}^nC_n u . D^n v)$$

Ques 3(i)

Given $u = \log(x^3+y^3+z^3-3xyz) \longrightarrow (1)$

on diff. u partially w.r.t x, y and z, we get

$$\frac{\partial u}{\partial x} = (3x^2-3yz)/(x^3+y^3+z^3-3xyz) \longrightarrow (2)$$ +1

$$\frac{\partial u}{\partial y} = (3y^2-3zx)/(x^3+y^3+z^3-3xyz) \longrightarrow (3)$$ +1

$$\frac{\partial u}{\partial z} = (3z^2-3xy)/(x^3+y^3+z^3-3xyz) \longrightarrow (4)$$ +1

adding equ. (2) (3) and (4)

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3(x^2+y^2+z^2-xy-yz-zx)}{[(x^3+y^3+z^3)-(3xyz)]}$$ +1

$$= \frac{3(x^2+y^2+z^2-xy-yz-zx)}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}$$

i.e. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$ Answer +1

Q.3(ii)

Given $u = xy + \frac{a^3}{x} + \frac{a^3}{y} \longrightarrow (1)$

$\therefore \frac{\partial u}{\partial x} = y - \frac{a^3}{x^2}$; $\frac{\partial u}{\partial y} = x - \frac{a^3}{y^2}$ +2

For Max or Min put $\frac{\partial u}{\partial x} = 0$ and $\frac{\partial u}{\partial y} = 0$; we get

$\frac{\partial u}{\partial x} = 0 \Rightarrow y - \frac{a^3}{x^2} = 0 \Rightarrow a^3 = x^2y$ and $\longrightarrow (2)$

$\frac{\partial u}{\partial y} = 0 \Rightarrow x - \frac{a^3}{y^2} = 0 \Rightarrow a^3 = x.y^2 \longrightarrow (3)$

$\therefore x^2y = y^2x \Rightarrow x = y$ +0.5

put $x=y$ in eqn. (2) we get

$$x^3 = a^3 \Rightarrow x = a$$

$$\therefore y = a$$

$\therefore$ (a,a) is called stationary point.

Now $r = \frac{\partial^2 u}{\partial x^2} = \frac{2a^3}{x^3}$; $s = \frac{\partial^2 u}{\partial x \partial y} = 1$; $t = \frac{\partial^2 u}{\partial y^2} = \frac{2a^3}{y^3}$ +1.5

at the point (a,a)

$r = 2 > 0$; $s = 1$, $t = 2$

and $rt - s^2 = 2 \times 2 - (1)^2 = 3 > 0$

Since $rt - s^2 = 3 > 0$ and $r = 2 > 0$. Then u has a minima at (a,a) and $u_{min(a,a)} = a^2 + a^2 + a^2 = 3a^2$. +1

Answer

Que. 3(iii)

Euler Theorem If $f(x,y)$ is a homogeneous function of x and y with degree n; then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$ +1

Proof. since $f(x,y)$ is a homogeneous function of degree n then by definition

$$f(x,y) = x^n F\left(\frac{y}{x}\right) \longrightarrow (1)$$ +1

Diff. eqn. (1) partially wrt 'x' we get

$$\frac{\partial f}{\partial x} = x^n f'\left(\frac{y}{x}\right) \cdot \left(-\frac{y}{x^2}\right) + f\left(\frac{y}{x}\right) \cdot n x^{n-1}$$

$$\Rightarrow \frac{\partial f}{\partial x} = -y x^{n-2} f'\left(\frac{y}{x}\right) + n x^{n-1} f\left(\frac{y}{x}\right)$$

$$\therefore x\frac{\partial f}{\partial x} = -y x^{n-1} f'\left(\frac{y}{x}\right) + n x^n f\left(\frac{y}{x}\right) \longrightarrow (2)$$ +1

And $\frac{\partial f}{\partial y} = x^n f'\left(\frac{y}{x}\right) \cdot \frac{1}{x}$

$$\therefore y\frac{\partial f}{\partial y} = y x^{n-1} f'\left(\frac{y}{x}\right) \longrightarrow (3)$$ +1

adding eqn. (2) and (3) we get

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = -y x^{n-1} f'\left(\frac{y}{x}\right) + n x^n f\left(\frac{y}{x}\right) + y x^{n-1} f'\left(\frac{y}{x}\right)$$ +1

i.e. $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$

Que. 4(i) To prove $\Gamma\frac{1}{2} = \sqrt{\pi}$

We know that by definition of gamma function

$$\Gamma n = \int_0^\infty e^{-t} t^{n-1} dt \; ; \longrightarrow (1) \; ; n>0$$ +0.5

putting $n = \frac{1}{2}$ in (1) we get

$$\Gamma\frac{1}{2} = \int_0^\infty e^{-t} t^{-1/2} dt$$

put $t = x^2 \Rightarrow dt = 2x dx$

$$\therefore \Gamma\frac{1}{2} = 2\int_0^\infty e^{-x^2} dx \longrightarrow (2)$$ +0.5

simi. $$\Gamma\frac{1}{2} = 2.\int_0^\infty e^{-y^2} dy \longrightarrow (3)$$ +0.5

By (2) and (3)

$$(\Gamma\tfrac{1}{2})^2 = 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} dx\,dy$$

changing into polar coordinates $x = r\cos\theta$; $y = r\sin\theta$ +0.5

$dx\,dy = r\,dr\,d\theta$, limit $\theta \to 0$ to $\frac{\pi}{2}$; $r \to 0$ to ∞.

$$\therefore (\Gamma\tfrac{1}{2})^2 = 4\int_{\theta=0}^{\pi/2}\left\{\int_{r=0}^\infty e^{-r^2} r\,dr\right\} d\theta$$ +0.5

Again put $r^2 = u \Rightarrow 2r\,dr = du$

$$= 4\int_{\theta=0}^{\pi/2}\left\{\int_0^\infty e^{-u} . \frac{du}{2}\right\} d\theta = 2\int_{\theta=0}^{\pi/2} (e^{-u})_0^\infty d\theta$$

$$= 2.\int_{\theta=0}^{\pi/2} d\theta = 2.[\theta]_0^{\pi/2} = \pi$$ $(\because e^{-\infty} = 0; e^{-0} = 1)$

+0.5

i.e. $(\Gamma\frac{1}{2})^2 = \pi \Rightarrow \boxed{\Gamma\frac{1}{2} = \sqrt{\pi}}$ Answer.

Que-4(ii) By definition of Gamma function

$$\Gamma m = \int_0^\infty e^{-t} t^{m-1} dt \; ; \; m>0 \longrightarrow (1)$$ +1

putting $t = x^2$, $dt = 2x dx$ we get

$$\Gamma m = 2\int_0^\infty e^{-x^2} x^{2m-1} dx$$

simi. $$\Gamma n = 2.\int_0^\infty e^{-y^2} y^{2n-1} dy$$ +1

$$\therefore \Gamma m . \Gamma n = 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} x^{2m-1} y^{2n-1} dx\,dy$$ +1

Changing into polar coordinates i.e. $x = r\cos\theta$; $y = r\sin\theta$ +1

$dx\,dy = r\,dr\,d\theta$ limit $\theta \to 0$ to $\frac{\pi}{2}$ $r \to 0$ to ∞

$$\Gamma m . \Gamma n = 4\int_{\theta=0}^{\pi/2}\int_{r=0}^\infty e^{-r^2} \cos^{2m-1}\theta . \sin^{2n-1}\theta \; r^{2m+2n-2+1} dr\,d\theta$$ +1

$$= 2\int_0^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta\, d\theta . 2\int_{r=0}^\infty e^{-r^2} r^{2(m+n)-1} dr$$ +1

$\overline{|m} \cdot \overline{|n} = \beta(m,n) \cdot \overline{|m+n}$ $\quad \left(\because 2\int_{\theta=0}^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta = \beta(m,n)\right)$ +1

Hence $\beta(m,n) = \dfrac{\overline{|m} \cdot \overline{|n}}{\overline{|m+n}}$ Hence Proved.

Que 4 (iii) Let $I = \int_0^2 x \cdot (8-x^3)^{1/3} \, dx$

put $x^3 = 8y$ i.e. $x = 2 \cdot y^{1/3}$

$\therefore \quad dx = \frac{2}{3} y^{-2/3} dy$ +1

Limits when $x=0$; $y=0$; when $x=2$, $y=1$, we get +1

$$I = \int_0^1 2y^{1/3} (8-8y)^{1/3} \frac{2}{3} y^{-2/3} dy$$

$$= \frac{8}{3} \int_0^1 y^{-1/3} (1-y)^{1/3} dy$$

$$= \frac{8}{3} \int_0^1 y^{\frac{2}{3}-1} (1-y)^{4/3-1} dy$$ +1

$$= \frac{8}{3} \beta\left(\frac{2}{3}, \frac{4}{3}\right)$$ $(\because$ By defi. of $B(m,n))$ +1

$$= \frac{8}{3} \cdot \frac{\overline{|\tfrac{2}{3}}\ \overline{|\tfrac{4}{3}}}{\overline{|\tfrac{2}{3}+\tfrac{4}{3}}} = \frac{8}{3} \frac{\overline{|\tfrac{2}{3}} \cdot \frac{1}{3} \cdot \overline{|\tfrac{1}{3}}}{\overline{|2}}$$ +1

$$= \frac{8}{9} \cdot \overline{|\tfrac{1}{3}} \cdot \overline{|1-\tfrac{1}{3}} = \frac{8}{9} \cdot \frac{\pi}{\sin\left(\frac{\pi}{3}\right)}$$ $\left(\because \overline{|n}\,\overline{|1-n} = \frac{\pi}{\sin n\pi}\right)$ +1

$$= \frac{8}{9} \cdot \frac{\pi}{\sqrt{3}/2}$$

$$= \frac{16\pi}{9\sqrt{3}}$$ +1

i.e. $\int_0^2 x(8-x^3)^{1/3} dx = \dfrac{16\pi}{9\sqrt{3}}$ Answer.

Que. 5(i) Let $I = \int_0^3 \int_0^2 \int_0^1 (x+y+z)\,dx\,dy\,dz$

$\therefore I = \int_0^3 \int_0^2 \left\{ \int_0^1 (x+y+z)\,dz \right\} dx\,dy$

$= \int_0^3 \int_0^2 \left[xz + yz + \frac{z^2}{2} \right]_0^1 dx\,dy$ +1

$= \int_0^3 \int_0^2 \left(x+y+\frac{1}{2} \right) dx\,dy = \int_0^3 \left\{ \int_0^2 \left(x+y+\frac{1}{2} \right) dy \right\} dx$ +1

$= \int_0^3 \left(xy + \frac{y^2}{2} + \frac{y}{2} \right)_0^2 dx = \int_0^3 (2x+3)\,dx$ +1

$= \left[\frac{2x^2}{2} + 3x \right]_0^3 = \left[x^2 + 3x \right]_0^3 = 9 + 9 = 18.$ Answer +1

Que. 5(ii) Let $I = \int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\,dy}{1+x^2+y^2}$

$= \int_0^1 \left[\int_0^{\sqrt{1+x^2}} \frac{dy}{(1+x^2)+y^2} \right] dx$ +1

$= \int_{x=0}^1 \left[\frac{1}{\sqrt{1+x^2}} \tan^{-1}\left(\frac{y}{\sqrt{1+x^2}} \right) \right]_0^{\sqrt{1+x^2}} dx$ +2

$\left\{ \because \int \frac{dy}{a^2+y^2} = \frac{1}{a} \tan^{-1}\left(\frac{y}{a}\right) \right.$

$= \int_{x=0}^1 \frac{1}{\sqrt{1+x^2}} \left(\tan^{-1} 1 - \tan^{-1} 0 \right) dx$ +1

$= \frac{\pi}{4} \int_0^1 \frac{dx}{\sqrt{1+x^2}}$ $\left(\because \tan^{-1} 1 = \frac{\pi}{4} \right.$ +1

$= \frac{\pi}{4} \left[\log x + \sqrt{1+x^2} \right]_0^1$ +1

$= \frac{\pi}{4} \log(1+\sqrt{2})$ or $\frac{\pi}{4} \sinh^{-1}(1)$. Answer +1

Q. 5(iii) Let $I = \iint_R y\,dx\,dy$; where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$

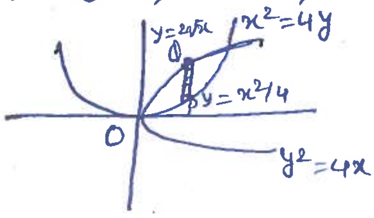
$\because y^2 = 4x \rightarrow (1)$ and $x^2 = 4y \rightarrow (2)$

solving (1) and (2) we get point of intersection

$\left(\frac{x^2}{4} \right)^2 = 4x \Rightarrow x(x^3 - 64) = 0 \Rightarrow x = 0, 4$ +1

i.e. Limit for $x \rightarrow 0$ to 4 +1 (for limit)

and for $y \rightarrow \frac{x^2}{4}$ to $2\sqrt{x}$ +1 (for figure)



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$$\therefore I = \int_{x=0}^{4} \int_{y=(x^2/4)}^{2\sqrt{x}} y\, dy\, dx$$ +1

$$= \int_{x=0}^{4} \left[\frac{y^2}{2}\right]_{x^2/4}^{2\sqrt{x}} dx = \int_{x=0}^{4} \left(4x - \frac{x^4}{16}\right) dx$$ +1

$$= \frac{1}{2}\left[2x^2 - \frac{x^5}{80}\right]_0^4 = \frac{48}{5}$$ Answer +1

Que.6(i) Given $(1+4xy+2y^2)dx + (1+4xy+2x^2)dy = 0 \longrightarrow (1)$

On comparing with $Mdx + Ndy = 0$

$M = (1+4xy+2y^2) \qquad N = (1+4xy+2x^2)$ +1

$\frac{\partial M}{\partial y} = 4x+4y \qquad \frac{\partial N}{\partial x} = 4y+4x$ +1

Clearly $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ i.e. given equation is exact.

The sol. of eqn. (1) is

$\int\limits_{y\text{-constant}} M.dx + \int (N, \text{not containing } x)\, dy = c$

$\int (1+4xy+2y^2)dx + \int 1\, dy = c$

$x + 2x^2y + 2xy^2 + y = c$ +1

or $x + y + 2x^2y + 2xy^2 = c$. Answer

Que6(ii) $(1+y^2) + (x - e^{-\tan^{-1}y})\frac{dy}{dx} = 0$

We can write $\frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{-\tan^{-1}y}}{1+y^2} \longrightarrow (1)$; which is Linear in x. +1

On comparing with $\frac{dx}{dy} + Px = Q$; Here $P = \frac{1}{(1+y^2)}$ $Q = \frac{e^{-\tan^{-1}y}}{1+y^2}$ +1

$\therefore$ I.F. $= e^{\int P dy} = e^{\int \frac{1}{(1+y^2)} dy} = e^{\tan^{-1}y}$ +1

The General sol. of (1) is given by $x.(\text{I.F.}) = \int (Q \times \text{I.F})\, dy + c$ +1

$\Rightarrow x.e^{\tan^{-1}y} = \int \frac{e^{-\tan^{-1}y}}{(1+y^2)} . e^{\tan^{-1}y}\, dy + c$ +1

$\Rightarrow x\, e^{\tan^{-1}y} = \int \frac{dy}{1+y^2} + c \Rightarrow x\, e^{\tan^{-1}y} = \tan^{-1}y + c$. +2

Answer

Que6(iii) $(D^2 - 2D + 5)y = e^{2x}\sin x$ +2

A.E. $m^2 - 2m + 5 = 0 \Rightarrow m = 1 \pm 2i$ +1

C.F. $= e^x[C_1\cos 2x + C_2 \sin 2x]$

P.I. $= \frac{1}{f(D)} Q = \frac{1}{(D^2-2D+5)} e^{2x}.\sin x$

$= e^{2x}.\frac{1}{((D+2)^2 - 2(D+2) + 5)} \sin x$ +1

$= e^{2x} \frac{1}{(D^2+2D+5)} \sin x = e^{2x}. \frac{\sin x}{((-1^2)+2D+5)}$

$= e^{2x}.\frac{1}{(2D+4)}\sin x = \frac{e^{2x}}{(2D+4)}.\frac{(2D-4)}{(2D-4)}\sin x$ +1

$\boxed{\text{P.I} = \frac{e^{2x}(\cos x - 2\sin x)}{10}}$ +1

General sol. $y = \text{CF} + \text{PI}$. +1