Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2018

AU3CO08/ FT3CO08 Fluid Mechanics

Programme: B.Tech. Branch/Specialisation: AU/FT

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Dimensions of a dynamic viscosity **1**

(a) MLT (b) $ML^{-1}T^{-2}$ (c) $ML^{-1}T^{-1}$ (d) $ML^{-2}T^{-2}$

ii. Capillary action is due to **1**

(a) Adhesion of liquid particles to a surface

(b) Cohesion of liquid particles

(c) Adhesion and cohesion

(d) Surface tension

iii. The continuity equation represents conservation of **1**

(a) Mass (b) Energy (c) Momentum (d) Vorticity

iv. A stream function **1**

(a) Satisfy Laplace equation for rotational motion

(b) May not remains constant for streamline

(c) Is a mathematical function, which has no physical equivalent

(d) Is defined only for steady and incompressible flow

v. A pitot with coefficient of velocity is unity used to measure the velocity of water. The differential manometer reading 10mm of liquid column with relative density of 10. The velocity of water in m/s. **1**

(a) 0.09 (b) 90 (c) 132 (d) 1.32

vi. A jet of water issues from a nozzle with a velocity 20 m/s impinges normally on flate plate, moving away from it at 10 m/s, the cross sectional area of the jet is 0.01m^2. The force developed on the plate is **1**

(a) 1000N (b) 100N (c) 10N (d) 2000N

P.T.O.

vii. Mach number is defined as the ratio of **1**
(a) Inertia force to viscous force
(b) Inertia force to elastic force
(c) Viscous force to surface tension
(d) Viscous force to elastic force

viii. Dynamic similarity between model and prototype means **1**
(a) Similarity of forces (b) Similarity of motion
(c) Similarity of shape (d) None of these

ix. The maximum velocity of one dimensional incompressible fully developed viscous flow between two fixed parallel plates is 6m/s. The mean velocity of the flow is **1**
(a) 2 (b) 3 (c) 4 (d) 5

x. A flow is said to be laminar when **1**
(a) The fluid particles moves in zigzag way
(b) The fluid particles move in layers parallel to the boundary
(c) The Reynolds number is high
(d) None of these

Q.2 i. Define specific weight and specific gravity? **2**
ii. Oil used for lubrication between shaft and sleeve having viscosity is 6 poise. The shaft of diameter 0.4m rotates at 190 RPM. The thickness of oil film is 1.5mm. calculate the power lost in bearing for a sleeve length of 90mm. **8**
OR iii. Find density of metallic body which floats at interface of mercury having specific gravity 13.6 and water such that 40% of its volume is submerged in mercury and 60% in water. **8**

Q.3 i. Define steady and non uniform flow with suitable example. **3**
ii. For a steady flow, the velocity field is given by $V = (-x^2 + 3y)i + (2xy)j$. Find the magnitude of acceleration of a particle at (1, -1). **7**
OR iii. Derive an expression for continuity equation in three dimensional flow. **7**

Q.4 i. State the Bernoulli's theorem with assumptions. Also write some important practical application of Bernoulli's equation. **4**

ii. Derive an expression for rate of flow through venturimeter. **6**
OR iii. A venturimeter of 20mm throat diameter is used to measure the velocity of water in a horizontal pipe of 40mm diameter. If the pressure difference between the pipe and throat section is found to be 30kpa. Find the flow velocity at inlet and throat. Assume frictional losses are negligible. **6**

Q.5 i. State and explain Buckingham π-theorem. **3**
ii. Prove that the Reynolds number is $\rho vd/\mu$. Where ρ is the density of fluid, v is the velocity of fluid, μ is the dynamic viscosity of fluid and d is the diameter of pipe. **7**
OR iii. Water flows through a pipe having an inner radius of 10mm at the rate of 36 kg/hr at 25 ^{0}C. The viscosity of water at 25 ^{0}C is 0.001kg/metre-sec. Find the Reynolds number of the flow. **7**

Q.6 i. Explain the laminar and turbulent flow with examples. **3**
ii. A laminar flow is taking place in a pipe of diameter 200mm. The maximum velocity is 1.5m/s. Find the mean velocity and radius at which mean velocity occurs. **7**
OR iii. Derive an expression for velocity distribution for viscous fluid flowing through a circular pipe. **7**

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Q(1) (i) c) $ML^{-1}T^{-1}$

(ii) c) Adhesion and cohesion

(iii) a) mass

(iv) d) is defined only for steady & incompressible flow.

(v) d) 1.32 m/s

(vi) b) 100 N

(vii) b) inertia force to elastic force

(viii) a) Similarity of forces

(ix) c) 4

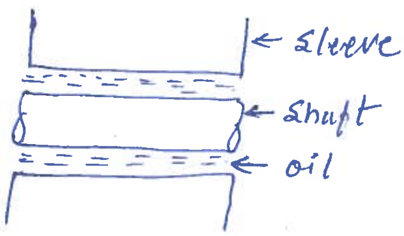
(x) b) The fluid Particles move in layers parallel to the boundary

Q(2).

Q2/(i) Sp. wt $(w) = \frac{\text{wt of fluid}}{\text{vol of fluid}} = \rho g$. (1)

Sp. gravity$(s) = \frac{\text{wt density or density of fluid}}{\text{wt. density or density of standard fluid}}$ (1)

Q2/(ii)



given data:-

$\mu = 6$ poise $= 0.6\ \frac{Ns}{m^2}$

dia of Shaft $d = 0.4$ m.

Speed of Shaft $N = 190$ RPM.

thickness of oil film $dy = 1.5$ mm $= 1.5 \times 10^{-3}$ m.

Sleeve length $L = 90$ mm $= 90 \times 10^{-3}$ m.

(1)

Tangential velocity of Shaft $u = \frac{\pi d N}{60} = 3.98$ m/s. (2)

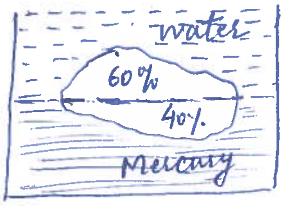
Shear stress $\tau = \mu\left(\frac{\partial u}{\partial y}\right) = \left(\frac{6}{10}\right)\left(\frac{3.98}{1.5 \times 10^{-3}}\right) = 1592\ \frac{N}{m^2}$ (2)

Shear Force on Shaft $F = (\tau \times A) = 1592 \times (\pi d L) = 180.05$ N (1)

Torque on the Shaft $T = F \times \frac{d}{2} = 36.01$ N-m (1)

Power lost $P = \frac{2\pi N T}{60} = 716.48$ watt Ans. (1)

Q(2)/(ii)



Let V = total vol. of the body

vol. of body submerged in mercury $= 0.4V$

vol. of body submerged in water $= 0.6V$ 1

$\Rightarrow$ total buoyant force = wt. of the body

$\Rightarrow$ (F_B due to water + F_B due to mercury) = wt. of the body 2

$$\Rightarrow (\rho V g)_{water} + (\rho V g)_{Hg} = (\rho V g)_{body} \quad 2$$

$$\Rightarrow \rho_w (0.6\cancel{V}) g + \rho_{Hg} (0.4\cancel{V}) g = \rho_b . \cancel{V} . \cancel{g} \quad 2$$

$$\Rightarrow 10^3 \times (0.6) + 13600 (0.4) = \rho_b$$

$$\Rightarrow \rho_b = 6040 \ kg/m^3. \quad \underline{Ans.} \quad 1$$

Q(3)/(i) <u>Steady Flow</u>:- when fluid parameters at any point in the flow field remains const. with respect to time. $\frac{\partial \rho}{\partial t} = const$ 1

Ex:- flow of water in pipe line due to centrifugal pump being run at uniform rotational speed. 0.5

<u>uniform flow</u>:- when fluid parameters (Pressure, velocity, density etc) remains const. with respect to space. $\frac{\partial \rho}{\partial s} = const.$ 1

Ex:- Flow at const. rate in a pipe line of const. cross-sectional area. 0.5

Q(3)/(ii) velocity vector is given by $\vec{v} = (-x^2+3y)\hat{i} + 2xy\hat{j}$

velocity in x-dirn $u = (-x^2+3y)$

velocity in y-dirn $v = 2xy$

acceleration vector $\vec{a} = (a_x\hat{i} + a_y\hat{j})$ — 1 1

$$\Rightarrow a_x = \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = (-x^2+3y)\frac{\partial}{\partial x}(-x^2+3y) + 2xy\frac{\partial}{\partial y}(-x^2+3y)$$

$$\Rightarrow a_x = (-x^2+3y)(-2x) + 2xy(3)$$

$\Rightarrow$ ~~a_x~~ at point $(1,-1)$

$$\Rightarrow a_x = [(-1-3)(-2) + (-2)(3)] = [8-6] = 2 \quad 2$$

$$\Rightarrow a_y = \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = (-x^2+3y)\frac{\partial}{\partial x}(2xy) + 2xy\frac{\partial}{\partial y}(2xy)$$

$$= (-x^2+3y)(2y) + 2xy(2x) \Rightarrow (-2x^2y + 6y^2 + 4x^2y)$$

$\Rightarrow a_y = (-2xy^2 + 6y^2 + 4x^2y)$

$\Rightarrow a_y = [-2\times(1) + 6\times1 + 4\times1\times(-1)]$

$\Rightarrow a_y = [+2+6-4] = 4$ ②

From Eq (1) $\vec{a} = (a_x i + a_y j)$

$\vec{a} = (2i + 4j)$

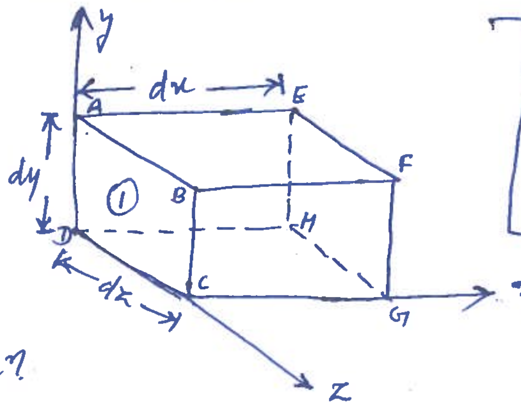
Magnitude of acceleration $|\vec{a}| = \sqrt{(2)^2 + (4)^2}$

$= \sqrt{4+16}$

$= 2\sqrt{5}$ Ans / 4.47 ②

Q(3)/(iii) Expression for continuity eqⁿ for 3-Dimensional flow:-

Consider an element of fluid of dimension dx, dy and dz in x, y and z-dirⁿ.



Suppose u is velocity in x-dirⁿ
v is velocity in y-dirⁿ
and w is velocity in z-dirⁿ

ρ be the density of flowing fluid and volume of the element is $(dx \cdot dy \cdot dz)$ ②

Mass of fluid Entering in section 1 (ABCD) per second

$= \frac{\rho \times vol.}{time} = \rho \times Q$ $\{Q = AV\}$

$= \rho(dy \cdot dz) \cdot u$ --- ①

Mass of fluid leaving from section 2 per second (EFGH) is given by:-

$\Rightarrow \rho(dy \cdot dz) + \frac{\partial}{\partial x}\rho(dy \cdot dz)u\,dx$ --- ②

Total gain of mass in x-dirⁿ = Eqⁿ(1) − Eqⁿ(2)

$= -\frac{\partial}{\partial x}\rho(dy \cdot dz)u\,dx$

Similarly gain of mass in y-dirⁿ $= -\frac{\partial}{\partial y}\rho(dx \cdot dz)v \cdot dy$

gain of mass in z-dirⁿ $= -\frac{\partial}{\partial z}\rho(dx \cdot dy)w \cdot dz$ ②

$\Rightarrow$ net increase of mass per unit time in fluid Element = rate of increase of mass in the element.

$$\Rightarrow -\left[\frac{\partial}{\partial x}(\rho u\, \cancel{dy\,dz\,dx}) + \frac{\partial}{\partial y}(\rho v\, \cancel{dx\,dy\,dz}) + \frac{\partial}{\partial z}(\rho w\, \cancel{dx\,dy\,dz})\right]$$

$$= \frac{\partial}{\partial t}(\rho \cdot \text{vol}) = \frac{\partial}{\partial t}(\rho\, \cancel{dx\,dy\,dz})$$ 3

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

it is the most general form of continuity Eq^n in Cartesian co-ordinates.

and this Eq^n is applicable to
A) Steady and unsteady flow
B) uniform & non-uniform flow
c) Compressible & incompressible flow.

For steady and incompressible Flow:-

$$\boxed{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0}$$

Q(4)/(i) it states that in a steady, ideal flow of an incompressible fluid, the total Energy at any point of the fluid is constant.
The total Energy consist of pressure Energy, Kinetic Energy and potential Energy or datum Energy.

2 Mathematically $\left[\frac{P}{\rho g} + \frac{V^2}{2g} + z\right] = \text{const}$

- z → datum Energy
- $\frac{V^2}{2g}$ → Kinetic Energy
- $\frac{P}{\rho g}$ → pressure Energy.

Assumptions:-

1 A) fluid is ideal, viscosity is zero
B) the flow is steady
c) The flow is incompressible
D) Flow is irrotational.

1 Application:- 1) Venturimeter
2) orifice meter
3) pitote tube.

Q(4)/(ii) Expression For Rate of Flow:-

(1) Venturimeter:- it is a device used for finding out discharge in a pipe.

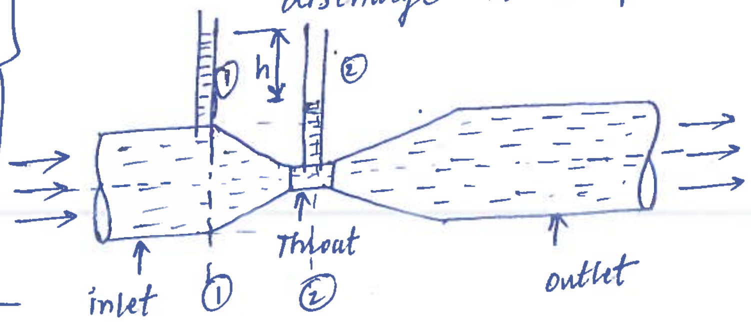


fig:- venturimeter

(2) Consider fluid is flowing through venturimeter. A Manometer is attached (one limb at inlet and other at throat)

Suppose a_1 be the inlet area of pipe and

a_2 be the area of throat

h be diff. of level of fluid in manometer

Apply Bernoullis Eqn for point 1 (inlet) and point 2 (throat)

$$\Rightarrow \left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1\right) = \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2\right) \quad -(1)$$

For same datum $Z_1 = Z_2$

and Eqn (1) becomes

$$\Rightarrow \frac{(P_1 - P_2)}{\rho g} = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) \quad -(2)$$

(3) From continuity Eqn $a_1 V_1 = a_2 V_2$

$$V_2 = \frac{a_1 V_1}{a_2}$$

From Eqn (2) $\Rightarrow$ $$\frac{(P_1 - P_2)}{\rho g} = \left(\frac{a_1^2 V_1^2 / a_2^2}{2g} - \frac{V_1^2}{2g}\right)$$

$$\Rightarrow \frac{(P_1 - P_2)}{\rho g} = \frac{V_1^2}{2g}\left(\frac{a_1^2}{a_2^2} - 1\right)$$

$$\Rightarrow \frac{(P_1 - P_2)}{\rho g} = \frac{V_1^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

$$\Rightarrow h = \frac{V_1^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_2^2}\right)$$

$$\Rightarrow V_1 = \frac{a_2\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

Rate of flow $Q = a_1 V_1 = a_2 V_2$

$\Rightarrow \quad Q = a_1 V_1$

$$\Rightarrow \quad Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$\Rightarrow \quad \boxed{Q_{th.} = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}}$$

Q(4)/(iii) Soln:- given data: Throat dia $d_2 = 20$ mm.
inlet pipe dia $d_1 = 40$ mm.
pressure diff $\Delta P = (P_1 - P_2) = 30 \times 10^3$ Pa

2 From continuity eqn $\Rightarrow A_1 V_1 = A_2 V_2$

$\Rightarrow \quad V_2 = 4V_1$

Apply Bernoullis eqn for inlet and throat:-

$$\Rightarrow \quad \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

for same datum $Z_1 = Z_2$

2
$$\Rightarrow \quad \left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g}\right) = \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g}\right)$$

$$\Rightarrow \quad \frac{(P_1 - P_2)}{\rho} = \frac{(V_2^2 - V_1^2)}{2}$$

$$\Rightarrow \quad \frac{30 \times 10^3}{10^3} = \frac{(4V_1)^2 - V_1^2}{2}$$

2 $\Rightarrow \quad V_1 = 2$ m/s $\rightarrow$ inlet velocity
and $V_2 = 8$ m/s $\rightarrow$ velocity at throat. } Ans.

Q(5)/(i) Buckingham's π-Theorem:$\Rightarrow$

" if there are n variables (independent and dependent) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into (n-m) dimensionless terms. Each term is called π-term. 1.5

Q(5)(i) Method of selecting Repeating variables:-

no. of repeating variables are equal to the no. of fundamental dim^n of the problem.

1) As far as possible dependent variable should not be selected as repeating variables.

2) one variable should contain geometric property (Ex:- length L, diameter d, Height H)

other variable contains flow property (Ex:- velocity v, acceleration a)

and third variable contain fluid property (Ex:- density ρ, viscosity μ, sp. weight w).

(3) Repeating variables should not form a dimensionless group.

4) R.V. together must have the same no. of fundamental dimensions

5) No two repeating variables should have same dimensions.

(1) (0.5)

Q(5)/(ii) Reynold's number $Re = \frac{\text{inertia force}}{\text{viscous force}} = \frac{F_i}{F_v}$ —(1) (1)

$$\text{inertia force } F_i = \text{mass} \times \text{acc. of flowing fluid}$$
$$= (\rho \times vol.) \times \frac{\text{velocity}}{\text{Time}}$$
$$= \rho \times \frac{vol}{Time} \times \text{velocity}$$
$$= \rho A V^2$$

(2)

$$\text{viscous force} = (\text{shear stress} \times \text{Area})$$
$$= \tau \times A$$
$$= \left(\mu \frac{du}{dy}\right) \cdot A = \mu \frac{V}{L} \cdot A$$

(2)

from Eq^n(1)st $Re = \frac{\rho A V^2}{\mu V A / L} = \frac{\rho \cancel{A} V^{\cancel{2}} L}{\mu \cancel{V} \cancel{A}} = \frac{\rho V L}{\mu}$ (2)

in case of pipe flow L will taken as d (dia of pipe)

$\boxed{Re = \frac{\rho V d}{\mu}}$ hence proved.

Q(5)/(iii):→ given inner radius $r = 10\,mm$, so $d = 20\,mm$ or $0.02\,m$

flow rate $\dot{m} = 36\ kg/hr = 0.01\ kg/sec$ & $\mu = 0.001\ kg/ms$ (2)

from continuity Eq^n $\dot{m} = \rho A V$ → $V = \frac{\dot{m}}{A\rho} = \frac{4\dot{m}}{\rho \pi d^2}$ (2)

and Reynolds no. $Re = \frac{\rho V d}{\mu} = \frac{\rho d}{\mu}\left(\frac{4\dot{m}}{\rho \pi d^2}\right) = \frac{4\dot{m}}{\mu \pi d}$ (2)

$Re = 636.94$ Ans. (1)

Q(6)/(i) Laminar Flow:- Fluid particles moves along a well-defined path. or in a straight line. when $Re < 2000$ } 1.5

Ex:- flow of fluid in a pipe at low velocity and highly viscous.

Turbulent Flow:- flow in which fluid particles moves in zig-zag way or Random manner. when $Re > 4000$.

Ex:- flow of fluid in a large dia pipe with high velocity and less viscous } 1.5

Q6/(ii) given data $D = 200\,mm = 0.2\,m$, $U_{max} = 1.5\,m/s$ } 1

i) Mean velocity $\bar{u}$ ⇒ $\frac{U_{max}}{\bar{u}} = 2$ or $\bar{u} = 0.75\,m/s$ } 2

ii) Radius at which $\bar{u}$ occurs:-

$$\Rightarrow \bar{u} = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)(R^2 - r^2)$$

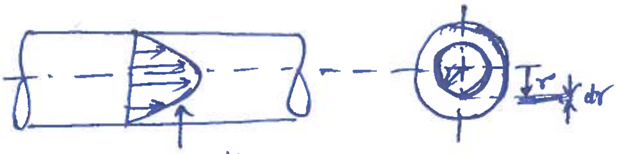
$$\Rightarrow \bar{u} = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2\left[1 - \frac{r^2}{R^2}\right] \quad \text{} 2$$

$$\Rightarrow \bar{u} = U_{max}\left[1 - \frac{r^2}{R^2}\right] \quad \} 1$$

$$\Rightarrow 0.75 = 1.5\left[1 - \frac{r^2}{(0.1)^2}\right] \Rightarrow \left[1 - \frac{r^2}{(0.1)^2}\right] = 0.5 \quad \} 1$$

$$\Rightarrow r = 0.0707\,m = 70.7\,mm \quad \underline{Ans.}$$

Q6/(iii)



velocity distribution

1

{ $y = R - r$, $dy = -dr$ }

{ y is measured from pipe wall }

Shear stress in the pipe $\tau = \mu\left(\frac{du}{dy}\right)$

$$\Rightarrow \tau = \mu\left(-\frac{du}{dr}\right) \quad \text{--- } 1$$

Shear stress in pipe is given by $\tau = -\frac{\partial P}{\partial x}\cdot\left(\frac{r}{2}\right)$ } 1

$$\Rightarrow \cancel{-}\mu\frac{du}{dr} = \cancel{-}\frac{\partial P}{\partial x}\left(\frac{r}{2}\right) \Rightarrow \frac{du}{dr} = \frac{1}{\mu}\cdot\frac{\partial P}{\partial x}\left(\frac{r}{2}\right)$$

$$\Rightarrow \text{on integration} \Rightarrow u = \frac{1}{4\mu}\cdot\frac{\partial P}{\partial x}r^2 + c \quad \text{--- } 2 \quad \} 2$$

c is called const. of integration and for its value boundary condition

at $r = R$, $u = 0$ ⇒ $c = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2$

From Eqn (2) ⇒ $u = \frac{1}{4\mu}\cdot\frac{\partial P}{\partial x}r^2 - \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2$

$$\Rightarrow \boxed{u = \frac{1}{4\mu}\cdot\frac{\partial P}{\partial x}\left[R^2 - r^2\right]}$$ hence prooved. } 2